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The singular behavior of the temperature gradient in the vicinity of a macrocrack tip

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Abstract—The power of singularity of the temperature gradient at a macrocrack tip is analyzed in this work. For a crack in an infinite medium. Williams' method of eigenfunction expansions is extended to heat conduction problems with a crack and comparison with the complex function approach is made. The intensity factor of temperature gradient (IFTG) is introduced to quantify the thermal energy cumulated in the neighborhood of a macrocrack tip. As an entirety, the power of singularity of the temperature gradient is analyzed for a crack in both isotropic and orthotropic media, and an interfacial crack between dissimilar materials. It is shown that the power of singularity of the temperature gradient is not affected by the discontinuous jumps of the thermal properties across the material interface, while that for a crack in an orthotropic medium depends on the ratio of thermal conductivities in the principal directions of material orthotropy.

INTRODUCTION

THE CONCEPT of damage tolerance is widely used in structural design which evaluates the structural performance in transferring loads under crisis situations. A macrocrack is one of the most popular mechanisms considered for this purpose and the load-bearing capacity of the material in the neighborhood of the crack tip is measured by the stress intensity factor [1]. For layered composites with various stacking sequences of the material layers, for example, the optimal stacking sequence is determined such that the stress intensity factor at the crack tip under the same geometrical and loading conditions attains the minimum value among all the possible cases with different stacking sequences. Another approach employing the deformation energy in the near-tip area to serve the same purpose has been discussed in ref. [2].

When a crack tip is closely approached, due to the abrupt change of the geometrical curvature, both the stresses and the strain energy density approach infinity. For an elastic solid, the near-tip stress behaves as $1/\sqrt{r}$, with r being the radial distance measured from the crack tip, while the strain energy density $\mathrm{d}W/\mathrm{d}V$ behaves as $1/r$ [3]. The stress intensity and the strain energy density factors are defined as

$$K = \lim_{r \to 0} \sqrt{r}\sigma_{\theta\theta} \quad \text{and} \quad S = \lim_{r \to 0} r(\mathrm{d}W/\mathrm{d}V) \qquad (1)$$

with $\sigma_{\theta\theta}$ being the circumferential stress component. Clearly, the concept of the intensity factors K and S relies upon the singular behavior of the near-tip stress and energy density.

In a parallel philosophy to the damage tolerance concept in solid mechanics, the *energy* bearing capacity of a solid medium could be assessed by the singular behavior of the temperature gradient in the vicinity of a crack tip. According to a previous analysis by Sih [4], the near-tip temperature in a homogeneous Fourier's solid behaves as $\sqrt{r}$ and the temperature gradient in the radial direction consequently behaves as $1/\sqrt{r}$. If we define the intensity factor of the temperature gradient (IFTG) at the crack tip in the same manner as those in equation (1)

$$\text{IFTG} = \lim_{r \to 0} 2\sqrt{r}T_{,r} \qquad (2)$$

a solid with better energy-bearing capacity should be the one possessing a lower value of IFTG under the same geometrical and loading conditions. Obviously, such a concept lies again in the singular behavior of the temperature gradient as the crack tip is closely approached.

Bearing these observations in mind, the present work aims at the derivation of the singular behavior of the temperature gradient in the vicinity of a macrocrack tip. In addition to that in an infinite medium, the present study also includes an interfacial crack between dissimilar materials and the effect of material orthotropy. A method of eigenfunction expansion developed by Williams [5] and extended recently in ref. [6] will be used in the analysis to determine the power of singularity and the angular distribution of the temperature gradient in the near-tip region without attributing to the solution of dual integral equations [1] or the complex potential functions [4].

ANALYSIS

(a) *A crack in an infinite medium*

In making contact with the analysis employing the complex potential functions [4], we first consider a crack in an infinite medium subject to remote heat

NOMENCLATURE

- a half length of the crack [m]
- C_{in} coefficients in the series solutions, $i = 1$–4
- F_n eigenmode for the angular distribution of temperature
- k thermal conductivity [$\mathrm{W\,m^{-1}\,K^{-1}}$]
- k_R ratio of the principal values of the orthotropic thermal conductivity, k_r/k_θ
- n integers from 0 to ∞
- q_0 heat flux applied remotely [$\mathrm{W\,m^{-2}}$]
- K stress intensity factor [$\mathrm{Pa\,m^{1/2}}$]
- r radial distance measured from the crack tip [m]
- S strain energy density factor [$\mathrm{J\,m^{-2}}$]
- T temperature [K].

Greek symbols
- θ polar angle [deg]
- λ_n eigenvalues, $n = 0, 1, 2, \ldots$
- σ stress components [Pa].

Superscript and subscripts
- $X_{,\eta}$ $\partial X/\partial \eta$ with $\eta = r, \theta$
- $X^{(i)}$ physical quantity X in the material layer i, $i = 1, 2$
- X_η component of X in the η-direction with $\eta = r, \theta$.

fluxes q_0 as shown in Fig. 1. The half length of the crack is denoted by a and the solid medium in the first case is assumed to be homogeneous and isotropic. The temperature at the crack surface is kept at a constant value which is assumed to be zero without loss in generality. With respect to the polar coordinates (r, θ) centered at the crack tip, the energy equation under a steady state is simply a Laplace equation

$$\nabla^2 T(r,\theta) = T_{,rr} + (1/r)T_{,r} + (1/r^2)T_{,\theta\theta} = 0 \quad (3)$$

and the boundary conditions to be satisfied in the near-tip region are

$$T = 0 \quad \text{at } \theta = \pm\pi. \quad (4)$$

The problem formulated in this manner clearly displays an eigenvalue problem. Because the Laplace equation is homogeneous with respect to the radial distance r, a product form for T satisfying equation (3) can be assumed

$$T(r,\theta) = \sum_{n=0}^{\infty} r^{\lambda_n+1} F_n(\theta) \quad (5)$$

with λ_n being the eigenvalues to be determined. Substituting equation (5) into equation (3), a second-order ordinary differential equation governing the function $F_n(\theta)$ is obtained

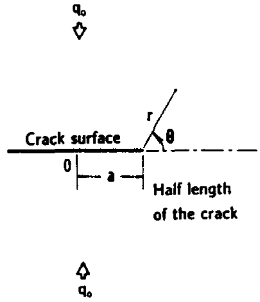

FIG. 1. A macrocrack with length $2a$ in an infinite medium subject to symmetrical heat fluxes applied remotely.

$$F_{n,\theta\theta} + (\lambda_n + 1)^2 F_n = 0 \quad (6)$$

which can be integrated directly to give

$$F_n(\theta) = C_{1n}\cos[(\lambda_n+1)\theta] + C_{2n}\sin[(\lambda_n+1)\theta]. \quad (7)$$

From boundary conditions (4), two algebraic equations for C_{1n} and C_{2n} are rendered

$$C_{1n}\cos[(\lambda_n+1)\pi] + C_{2n}\sin[(\lambda_n+1)\pi] = 0$$
$$C_{1n}\cos[(\lambda_n+1)\pi] - C_{2n}\sin[(\lambda_n+1)\pi] = 0 \quad (8)$$

which gives the following eigenequation for the existence of a non-trivial solution:

$$\sin[2(\lambda_n+1)\pi] = 0 \quad \text{or } \lambda_n + 1 = n/2, \quad \text{for } n = 0, 1, 2, \ldots \quad (9)$$

Substituting equation (9) into equation (7) and the result into equation (5), the temperature $T(r, \theta)$ is

$$T(r,\theta) = \sum_{n=0}^{\infty} r^{n/2}[C_{1n}\cos(n\theta/2) + C_{2n}\sin(n\theta/2)] \quad (10)$$

where we can show from equations (8) and (9) in a straightforward manner that $C_{1n} = 0$ for n being even and $C_{2n} = 0$ for n being odd. Although the remote heat flux applied to the solid is not incorporated in equation (10), it provides sufficient information as far as the near-tip temperature gradient $T_{,r}$ is concerned. By taking the derivative with respect to r on equation (10) and expanding the resulting series, we have

$$T_{,r}(r,\theta) = \frac{C_{11}}{2\sqrt{r}}\cos(\theta/2) + r^0[C_{22}\sin\theta] + r[(3/2)C_{13}\cos(3\theta/2)] + O(r^{n/2}) \quad \text{for } n \geqslant 2. \quad (11)$$

In the vicinity of the crack tip with r approaching zero, clearly, the first term in equation (11) dominates and asymptotically

$$T_{,r}(r,\theta) \simeq \frac{C_{11}}{2\sqrt{r}}\cos(\theta/2) \quad \text{as } r \to 0. \quad (12)$$

Clearly, the temperature gradient presents a $1/\sqrt{r}$-type of singularity at the crack tip. The power of singularity of the near-tip temperature gradient is thus 1/2.

In summary, Williams' method illustrated above reduces the crack problem from a boundary value problem to an eigenvalue problem. The eigenvalues obtained in this manner reveal the r-dependency of the thermal field while the eigenfunctions relate to the angular distributions of the thermal field around the crack tip. The amplitude of the eigenfunction such as the coefficient C_{11} in equation (12), however, cannot be determined in general by the method. This is the biggest disadvantage of the method in comparison with the dual-integral equation and the complex function methods where the thermal field in the near-tip region can be determined as an entirety. It can be seen clearly that the remote boundary condition of q_0, for example, cannot be taken into account in the present approach. The method thus implies the same singular behavior of the temperature gradient in the near-tip region for the cracked solid subject to a remote heat flux q_0 or a temperature gradient $T^0_{,y}$.

Nevertheless, the coefficient C_{11} could be determined from the consideration of dimensional consistency under limited conditions. As shown by the configuration of the cracked solid in Fig. 1, the coefficient C_{11} should be a function of the half crack length a, the thermal conductivity k, and the heat flux q_0 applied remotely. This observation essentially leads to $C_{11} = q_0\sqrt{a}/k$ which has dimensions of deg $m^{-1/2}$. For the same cracked solid subject to a remote temperature gradient $T^0_{,y}$, as another example, the coefficient C_{11} takes the form of $\sqrt{a}(T^0_{,y})$ from the same consideration. With these expressions of C_{11}, the results shown by equation (12) are the same as those obtained by Sih [4] employing the Muskhelishvili formulation of the analytic complex functions. In front of the crack tip at $\theta = 0$, the temperature gradient $T_{,r}$ reaches a maximum value and the intensity factor of the temperature gradient in the two cases can be obtained as

$$\mathrm{IFTG} = \lim_{r\to 0} 2\sqrt{r}T_{,r} = C_{11} = \begin{cases} q_0\sqrt{a}/k, & \text{for the case of remote heat flux} \\ (T^0_{,y})\sqrt{a}, & \text{for the case of remote temperature gradient.} \end{cases} \quad (13)$$

(b) *An interfacial crack between dissimilar materials*

By extending the method of eigenfunction expansions adopted in (a), the singular behavior of the near-tip temperature gradient can be analyzed for an interfacial crack between dissimilar materials, as shown by Fig. 2. The thermal conductivity for the two contact material layers is respectively $k^{(1)}$ and $k^{(2)}$. Under a steady state, the temperature distribution in each material layer is represented by the combination of equations (5) and (7)

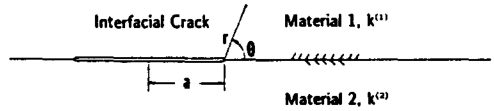

FIG. 2. An interfacial crack between dissimilar materials with thermal conductivities $k^{(1)}$ and $k^{(2)}$.

$$T^{(1)}(r,\theta) = \sum_{n=0}^{\infty} r^{\lambda_n+1}[C_{1n}\cos(\lambda_n+1)\theta + C_{2n}\sin(\lambda_n+1)\theta], \quad \text{for material 1}$$

$$T^{(2)}(r,\theta) = \sum_{n=0}^{\infty} r^{\lambda_n+1}[C_{3n}\cos(\lambda_n+1)\theta + C_{4n}\sin(\lambda_n+1)\theta], \quad \text{for material 2.} \quad (14)$$

Determination of the eigenvalues λ_n in this case depends on the following boundary conditions:

$$\begin{aligned} &T^{(1)} = 0 \quad \text{at } \theta = \pi, \quad \text{the top surface of the crack} \\ &T^{(2)} = 0 \quad \text{at } \theta = -\pi, \quad \text{the bottom surface of the crack} \\ &T^{(1)} = T^{(2)} \quad \text{and} \quad k^{(1)}T^{(1)}_{,\theta} = k^{(2)}T^{(2)}_{,\theta} \\ &\qquad \text{at } \theta = 0, \quad \text{the material interface.} \end{aligned} \quad (15)$$

Substituting equations (14) into equations (15) renders

$$\begin{aligned} &\sin[2(\lambda_n+1)\pi] = 0 \quad \text{or} \quad \lambda_n+1 = n/2, \quad \text{for } n = 0,\ 1,\ 2,\ldots \\ &C_{3n} = C_{1n}, \quad C_{4n} = [k^{(1)}/k^{(2)}]C_{2n}, \quad C_{1n} = 0 \\ &\qquad \text{for } n \text{ being even integers,} \\ &C_{2n} = 0 \quad \text{for } n \text{ being odd integers.} \end{aligned} \quad (16)$$

Note that the eigenvalues λ_n obtained in this case are the same as those obtained in the previous case, refer to equation (9), and consequently the r-dependency of the near-tip temperature and its gradient is not affected by the thermal properties of the contacting material layers. With equations (15) and (16), the asymptotic form of the temperature gradient in each material layer can be expanded to give

$$T^{(1)}_{,r} = \frac{C_{11}}{2\sqrt{r}}\cos(\theta/2) + r^0[C_{22}\sin\theta] + \sqrt{r}[(3/2)C_{13}\cos(3\theta/2)] + O(r^{n/2})$$

$$T^{(2)}_{,r} = \frac{C_{11}}{2\sqrt{r}}\cos(\theta/2) + r^0[C_{22}(k^{(1)}/k^{(2)})\sin\theta] + \sqrt{r}[(3/2)C_{13}\cos(3\theta/2)] + O(r^{n/2}) \quad (17)$$

with $n \geqslant 2$. In the near-tip region with r approaching zero, similarly, the asymptotic expressions for the temperature gradients are

$$T^{(1)}_{,r} \simeq \frac{C_{11}}{2\sqrt{r}}\cos(\theta/2) \quad \text{and} \quad T^{(2)}_{,r} \simeq \frac{C_{11}}{2\sqrt{r}}\cos(\theta/2) \quad \text{as } r \to 0. \quad (18)$$

Equation (18) has exactly the same form as equation

(12) for a crack in a homogeneous medium. It implies that the power of singularity at the interfacial crack tip is not affected by the thermal properties of the material layers in contact. Except for the involvement in the amplitude of C_{11}, the effect of the thermal conductivities, refer to equation (17), enters the *higher* order eigenmodes of the angular distribution of $T_{,r}^{(2)}$ which vanishes as r approaches zero. Due to the presence of two values of thermal conductivity in each material layer in this case, unfortunately, the coefficient C_{11} cannot be determined by the same consideration of dimensional consistency as that given in case (a). The thermal conductivity k involved in the first of equations (13) for the present case should be replaced by an effective value which is a combined function of both $k^{(1)}$ and $k^{(2)}$. Its exact form, however, cannot be determined from this approach.

(c) *A crack in an orthotropic material*

For a macrocrack in an orthotropic medium with k_r and k_θ being the principal values along the axes of material orthotropy, the energy equation can be written as

$$T_{,rr}+(1/r)T_{,r}+(k_R)^{-1}(1/r^2)T_{,\theta\theta}=0 \qquad (19)$$

with $k_R = k_r/k_\theta$. Assuming the same form as equation (5) for $T(r, \theta)$, the differential equation governing the function of $F_n(\theta)$ in this case takes the form*

$$F_{n,\theta\theta}+k_R(\lambda_n+1)^2F_n=0 \qquad (20)$$

which has the solution

$$F_n(\theta)=C_{1n}\cos[\sqrt{k_R}(\lambda_n+1)\theta]+C_{2n}\sin[\sqrt{k_R}(\lambda_n+1)\theta]. \qquad (21)$$

From the boundary conditions at the crack surfaces, equation (4), the eigenequation for the eivenvalues λ_n is

$$\sin[2\sqrt{k_R}(\lambda_n+1)\pi]=0 \quad \text{or } \lambda_n+1=n/2\sqrt{k_R}, \quad \text{for } n=0,1,2,\ldots \qquad (22)$$

and the coefficients $C_{1n}=0$ for n being even and $C_{2n}=0$ for n being odd. Equation (22) clearly indicates that the eigenvalues, and hence the r-dependency of the temperature gradient, depend on the ratio of the principal values of orthotropic thermal conductivity. Substituting the eivenvalues in equation (22) into the series for the temperature gradient, it follows that

$$T_{,r}=\frac{C_{11}}{2\sqrt{k_R}}r^{(1-2\sqrt{k_R})/2\sqrt{k_R}}\cos(\theta/2)+\frac{C_{22}}{\sqrt{k_R}}r^{(1-\sqrt{k_R})/\sqrt{k_R}}\sin(\theta)+\frac{3C_{13}}{2\sqrt{k_R}}r^{(3-2\sqrt{k_R})/2\sqrt{k_R}}\cos(3\theta/2)+\frac{2C_{24}}{\sqrt{k_R}}r^{(2-\sqrt{k_R})/\sqrt{k_R}}\sin(2\theta)+\cdots. \qquad (23)$$

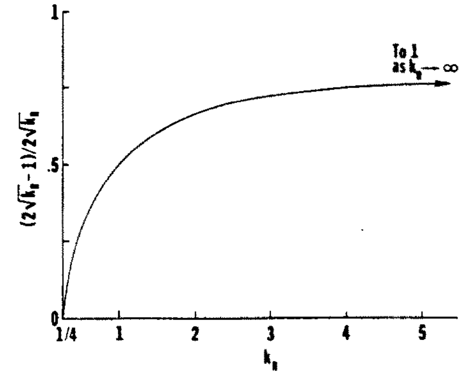

FIG. 3. The variation of the power of singularity of the near-tip temperature gradient $T_{,r}$ with respect to the ratio of $k_R(k_R = k_r/k_\theta)$.

The singular terms in equation (23) depend on the range of the conductivity ratio k_R. For $0 < k_R < 1/4$, all the terms are well behaved and the temperature gradient $T_{,r}$ approaches zero in the near-tip region with $r \to 0$. For $1/4 < k_R < 1$, the first term becomes singular while the other terms approach zero as r approaches zero. The asymptotic form of equation (23) in this case is thus

$$T_{,r}\simeq\frac{C_{11}}{2\sqrt{k_R}}r^{-(2\sqrt{k_R}-1)/2\sqrt{k_R}}\cos(\theta/2), \quad \text{as } r\to 0. \qquad (24)$$

In the range of $1 < k_R < 2.25$, the first two terms in equation (23) become singular but the first term diverges at a *faster* rate than the second term and the leading term in equation (23) again dominates in the near-tip region. This argument can be applied to the subsequent terms in equation (23) and it is informative to conclude that the near-tip temperature gradient is indeed represented by equation (24) in general.

In the present case with material orthotropy, the singularity of the temperature gradient exists at the crack tip only if

$$2\sqrt{k_R}>1 \quad \text{or } k_r/k_\theta>\tfrac{1}{4}. \qquad (25)$$

For the orthotropic material with k_R being smaller than 1/4, the temperature gradient approaches zero at the crack tip and no singularity exists. The power of singularity depicted by equation (24) is $(2\sqrt{k_R}-1)/2\sqrt{k_R}$. For an isotropic material with $k_r = k_\theta$, k_R reduces to a value of 1 and the power of singularity being 1/2 is retrieved. For strong directional materials with $k_r \gg k_\theta$, the ratio of k_R approaches infinity and the power of singularity has a limit value of 1. Figure 3 displays such a variation for the values of k_R being greater than 1/4. It shows that the power of singularity increases as the ratio of k_r/k_θ increases. The coefficient C_{11} in this case, similar to that in case (b), involves both k_r and k_θ and cannot be determined by the present approach.

CONCLUSION

The power of singularity of the temperature gradient provides a physical measure for the intensification of the thermal energy cumulated in the vicinity of a macrocrack tip. Three types of problems have been considered in the present analysis. They include a macrocrack in an infinite medium, an interfacial crack between dissimilar materials, and a crack in an orthotropic material. The first two problems present a power of singularity of the temperature gradient of 1/2, while the third presents a result depending on the ratio of the principal values of the orthotropic thermal conductivity k_r/k_θ. The power of singularity of the near-tip temperature gradient ranges from 0 to 1 as the ratio of k_r/k_θ varies. The near-tip temperature gradient is singular only if k_r/k_θ is greater than 1/4. Generally speaking, the heat conduction problem involving a crack presents a mixed type boundary value problem due to the different boundary conditions specified at the crack surface and along the crack line in front of the crack tip. A general solution for this type of problem can only be made by either the method of dual integral equations [1] or the complex potential function method [4]. As fas as the power of singularity and the fundamental modes of angular distributions of the temperature gradient around the crack tip are concerned, Williams' method of eigenfunction expansions does provide a more friendly approach. The method, however, cannot determine the amplitude of the angular distribution in a general sense.

For a crack propagating with a constant velocity v in a solid medium, the singular behavior of the temperature gradient at the crack tip can be complicated. With reference to the finite speed of heat propagation C in the solid, it has been shown [6] that the r-dependency of the temperature gradient in a homogeneous and isotropic medium intrinsically transits from $1/\sqrt{r}$, r^0, to r as the thermal Mach number M defined as v/C transits from the subsonic ($M < 1$), transonic ($M = 1$), to the supersonic ($M > 1$) ranges. The well-known $1/\sqrt{r}$-type of singularity exists only if the crack tip propagates at a speed *slower* than the thermal wave speed in the solid. When the crack tip velocity increases to the transonic and supersonic ranges with $M \geqslant 1$, the thermal energy does not have sufficient time to cumulate before the crack tip moves away, and the temperature gradient is bounded thereby. The thermal *shock* wave induced by a rapidly propagating crack tip is the central topic of research for this type of problems in lieu of the singular behavior of the temperature gradient.

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LE COMPORTEMENT SINGULIER DU GRADIENT DE TEMPERATURE AU VOISINAGE D'UNE MACROCRAQUELURE

Résumé—On analyse la singularisation du gradient de température près d'une fente de macrocraquelure. Pour une craquelure dans un milieu infini, la méthode des fonctions propres de Williams est étendues aux problèmes de conduction thermique avec craquelure et on fait la comparaison avec l'approche par fonction complexe. Le facteur d'intensité du gradient de température (IFTG) est introduit pour quantifier l'énergie thermique cumulée dans le voisinage de la lèvre de la macrocraquelure. On considère celle-ci dans des milieux isotropes et orthotropes, et aussi à l'interface entre deux matériaux différents. On montre que la puissance de singularité du gradient température n'est pas affectée par des sauts discontinus des propriétés thermiques à travers l'interface, tandis que pour une craquelure dans un milieu orthotrope cela dépend du rapport des conductivités thermiques dans les directions principales de l'orthotropie.

DAS SINGULÄRE VERHALTEN EINES TEMPERATURGRADIENTEN IN DER NÄHE EINER MAKRORISSPITZE

Zusammenfassung—In dieser Arbeit wird die Auswirkung eines singulären Temperaturgradienten an einer Makrorißspitze analysiert. Für einen Riß im unendlich ausgedehnten Medium wird das Verfahren der Entwicklung von Eigenfunktionen nach William ausgedehnt auf Wärmeleitprobleme in Anwesenheit eines Risses. Das Ergebnis wird mit dem Ansatz unter Verwendung komplexer Funktionen verglichen. Es wird der Intensitätsfaktor des Temperaturgradienten (IFTG) eingeführt, um ein Maß für die thermische Energie geben zu können, die sich in der Umgebung einer Makrorißspitze angesammelt hat. Zusätzlich wird die Stärke der Singularität des Temperaturgradienten für Risse in isotropen und orthotropen Medien analysiert, außerdem für einen Riß zwischen unterschiedlichen Materialien. Es zeigt sich, daß die Stärke der Singularität des Temperaturgradienten unabhängig von diskontinuierlichen Sprüngen der thermophysikalischen Eigenschaften an Materialgrenzen ist, im Fall eines Risses in einem orthotropen Medium hängt die Stärke dagegen vom Verhältnis der Wärmeleitfähigkeiten in der Hauptrichtung der Materialorthotropie ab.

СИНГУЛЯНОЕ ПОВЕДЕНИЕ ТЕМПЕРАТУРНОГО ГРАДИЕНТА ВБЛИЗИ ВЕРШИНЫ МАКРОТРЕЩИНЫ

Аннотация—Анализируется степень сингулярности температурного градиента вблизи вершины макротрещины. Для трещины в бесконечной среде метод разложений по собственным функциям, предложенный Вильясом, применяется к задачам теплопроводности и проводится его сравнение с подходом на основе комплексных функций. Для количественного определения тепловой энергии, накопленной вблизи вершины макротрещины, вводится коэффициент интенсивности температурного градиента (IFTG). Степень сингулярности температурного градиента анализируется в целом как для трещины в изотропных и ортотропных средах, так и для граничной трещины между разнородными материалами. Показывается, что степень сингулярности температурного градиента не зависит от скачкообразных изменений тепловых свойств на границе раздела материалов, но в случае трещины в ортотропной среде зависит от отношения коэффициентов теплопроводности в основных направлениях ортотропи материала.